Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–I & II(NEW)EXAMINATION – SUMMER 2022

Subject Code:3110014 **Date:02-08-2022**

Subject Name:Mathematics - 1

Time:10:30 AM TO 01:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Is $\sum_{n=1}^{\infty} \frac{1}{n^p}$ convergent for $p>1$? Justify your answer.	**03**
	(b)	(1) Find $\lim_{x\to a} \frac{\sin x - \sin a}{(x-a)^2}$	**02**
		(2) Is $\int_0^{\infty} \frac{\sin^2 x}{x^2}$ convergent? Justify your answer.	**02**
	(c)	(1) Find the length of curve $f(x)=\frac{x^3}{12}+\frac{1}{x}$, $1\le x\le 4$.	**04**
		(2) Prove that $Gamma(n)=(n-1)\,Gamma(n-1)$.	**03**
Q.2	**(a)**	Investigate the convergence of $\sum_{1}^{\infty} \frac{n^2}{7^n}$.	**03**
	(b)	Investigate the convergence of $\sum_{n=1}^{\infty} \frac{2^n (n!)^2}{(2n)!}$	**04**
	(c)	Find Fourier series of $f(x)=x^2$, $-\pi<x<\pi$.	**07**
		OR	
	(c)	Find Fourier series of $f(x) = x,\ -1<x<0$; $= 2,\ 0<x<1$.	**07**
Q.3	**(a)**	Find the derivative of $f(x,y)=x^2+xy+y^2$ in the direction $\hat{i}+\hat{j}$ at $P(1,1)$.	**03**
	(b)	Find the tangent plane of $z=e^x \cos y$ at $P(0,0,0)$.	**04**
	(c)	Find local extreme values of $f(x,y)=xy-x^2-y^2-x$.	**07**
		OR	
Q.3	**(a)**	Explain second derivative test for local extreme values.	**03**
	(b)	Let $f=\ln r$, where $\bar{r}=x\,\hat{i}+y\,\hat{j}+z\,\hat{k}$ and $r=\lvert\bar{r}\rvert$. Find $grad\ f$.	**04**
	(c)	Determine the minimum value of x^2yz^2 subject to the condition $x+y+2z=5$ using method of Lagrange multipliers.	**07**
Q.4	**(a)**	Evaluate $\int_{y=0}^{1}\int_{x=0}^{2} \frac{1}{\sqrt{4-x^2}\sqrt{1-y^2}}\,dxdy$.	**03**

(b) Evaluate the integral $\int_0^2 \int_{x/2}^1 \frac{1}{3} e^{y^2} dy\,dx$ **04**

by change of order.

(c) (1) Find the area of the region covered by $x=1,\ x=4,\ y=0$ and $y=\sqrt{x}$. **04**

(2) Evaluate $\int_{x=0}^{1} \int_{y=0}^{x^{1/4}} \int_{z=0}^{y^2} \sqrt{z}\ dz\ dy\ dx$ **03**

OR

Q.4 **(a)** Evaluate $\iint_R xy\ dA$ where R is the region bounded by x axis, the ordinate $x=2a$ and the curve $x^2=4ay$. **03**

(b) Evaluate the integral $\int_{y=0}^{1} \int_{x=0}^{\cos^{-1} y} e^{\sin x}\ dx\,dy$ by change of order. **04**

(c) (1) Change in to polar coordinates then solve $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-\left(x^2+y^2\right)} dy\ dx$. **04**

(2) Let $x+y=u$ and $y=uv$ are given transformations. Find Jacobian for change of variables. **03**

Q.5 **(a)** Find characteristic equation of $A=\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \\ 2 & 0 & 1 \end{bmatrix}$. **03**

(b) Find Maclaurin's series for $f(x)=e^{2x}\sinh x$ and show at least up to x^4 term. **04**

(c) Solve **07**

$x+y+w=1,\ 2x+z+w=3,\ 2y+z+2w=2$.

OR

Q.5 **(a)** Show that give matrix $A=\begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}$ satisfies its Characteristic equation. **03**

(b) Show that $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ converges. **04**

(c) Show that $A=\begin{bmatrix} 0 & 1 & 0 \\ -2 & -1 & 2 \\ -4 & -8 & 7 \end{bmatrix}$ is diagonalizable. Find the matrix of eigen vectors and diagonal matrix. **07**

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